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# Attitude Fault Tolerant Control of a Quadrotor UAV Using Robust Adaptive-Backstepping Method

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***Abstract*— This paper deals with the design of a fault tolerant control (FTC) of a quadrotor aircraft system, considering the actuator faults. The attitude dynamic model of quadrotor, taking into account the various parameters which can affect the dynamics of this system in space is presented. Subsequently, based on robust adaptive-backstepping method and taking into account the actuator faults, a new FTC strategy is developed. The main advantages of this control method are the good tracking and the stability preservation of the closed loop dynamics of quadrotor aircraft even after occurrence of actuator faults. Numerical simulation results are provided to show the good performances of proposed FTC method.**

***Keywords*— Adaptive control, Backstepping approach, Fault tolerant control (FTC), Robust control, Quadrotor.**

## I. Introduction

Quadrotors are one of Unmanned Aerial Vehicles (UAVs) which consist of two rods and four actuators as shown in Fig.1. Even though its structure is simple, the quadrotor is a VTOL (Vertical Task-off and Landing) and can perform most of missions that helicopters can do. In some aspects, the quadrotors have better maneuverability than helicopters because quadrotors have four rotors, which can increase the mobility and loadability. They, has been studied recently by some authors like [13], [20], [17], [7], [2], [10], [18], [19], [24], [1], [3], [5], [12], [21], [22], [9], [8], [6], [23], [4], [11]. These authors propose many other dynamics systems, present constant or slowly-varying uncertain parameters, but without considering the faults affecting these systems. However, in [15] and [16] the authors propose a control algorithms based on backstepping approach using sliding mode techniques, in order to allow the tracking of the various desired trajectories despite the occurrence of actuator faults. But, the corresponding inputs control of these control strategies are characterized by chattering phenomenon caused by the using of "sign" function.

In this paper, the attitude control problem of quadrotor aircraft in presence of actuator faults is considered. The dynamical model describing the quadrotor attitude motions, which contains the aerodynamics frictions, the gyroscopic effects, and the quadrotor moments, taking into consideration the actuator faults is presented in section II. Subsequently, based on backstepping approach, a fault tolerant control is developed, in which an adaptive algorithm is used to compensate the effects of actuator faults in quadrotor system. In section IV, all simulation results are summarized, when the proposed FTC scheme is applied to the quadrotor UAV. Finally, conclusions and futures advances are provided.

## II. Quadrotor Attitude Model

The quadrotor have four propellers in cross configuration. The two pairs of propellers {1,3} and {2,4} as described in Fig. 1, turn in opposite directions. By varying the rotor speed, one can change the lift force and create motion. Thus, increasing or decreasing the four propeller's speeds together generates vertical motion. Changing the 2 and 4 propeller's speed conversely produces roll rotation coupled with lateral motion. Pitch rotation and the corresponding lateral motion; result from 1 and 3 propeller's speed conversely modified. Yaw rotation is more subtle, as it results from the difference in the counter-torque between each pair of propellers.

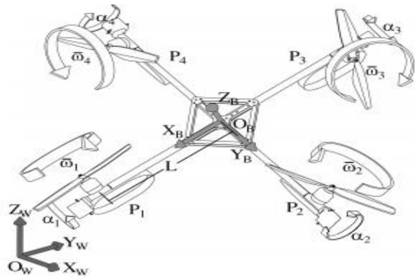


Fig. 1 Quadrotor configuration

The attitude dynamical model is represented by Euler angles $[\phi,\ \theta,\ \psi]^T$ corresponding to an aeronautical convention [14]. The attitude angles are respectively called Roll angle ($\phi$ rotation around x-axis), Pitch angle ($\theta$ rotation around y-axis) and Yaw angle ($\psi$ rotation around z-axis). It contains four terms which are the gyroscopic effect resulting from the rigid body rotation, and from the propeller rotation coupled with the body rotation, aerodynamics frictions and finally the quadrotor moments according to the body fixed frame [3], [5], [23], [4]:

$$\begin{cases} \ddot{\phi} = \dfrac{(I_y - I_z)}{I_x}\dot{\theta}\dot{\psi} - \dfrac{J_r}{I_x}\Omega_r\dot{\theta} - \dfrac{K_{fax}}{I_x}\dot{\phi}^2 + \dfrac{l}{I_x}u_1 \\ \ddot{\theta} = \dfrac{(I_z - I_x)}{I_y}\dot{\phi}\dot{\psi} + \dfrac{J_r}{I_y}\Omega_r\dot{\phi} - \dfrac{K_{fay}}{I_y}\dot{\theta}^2 + \dfrac{l}{I_y}u_2 \\ \ddot{\psi} = \dfrac{(I_x - I_y)}{I_z}\dot{\theta}\dot{\phi} - \dfrac{K_{faz}}{I_z}\dot{\psi}^2 + \dfrac{1}{I_z}u_3 \end{cases} \quad (1)$$

The system's inputs are posed $u_1$, $u_2$, $u_3$, and $\Omega_r$ is a disturbance, obtaining:

$$\begin{cases} u_1 = lb(\omega_4^2 - \omega_2^2) \\ u_2 = lb(\omega_3^2 - \omega_1^2) \\ u_3 = d\left(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2\right) \\ \Omega_r = \omega_1 - \omega_2 + \omega_3 - \omega_4 \end{cases} \quad (2)$$

The rotors are driven by DC motors with the well known equations [24]:

$$\begin{cases} J_r \dot{\omega}_i = \tau_i - Q_i \\ v_i = \dfrac{R_a}{k_m k_g}\tau_i + k_m k_g \omega_i \end{cases}, \; i \in \{1,2,3,4\} \quad (3)$$

where, $R_a$ is the motor resistance,
$k_m$ is the motor torque constant,
$k_g$ is the gear ratio.

## III. Fault Tolerant Control Algorithm of Quadrotor

The object of the FTC algorithm developed in this paper is to design a robust attitude tracking controller which makes the output of the system $\{\phi(t), \theta(t), \psi(t)\}$ to track the desired output $\{\phi_d(t), \theta_d(t), \psi_d(t)\}$ under actuator faults .

The complete model resulting by adding of actuator faults in dynamic model (1) can be written in a state-space form as

$$\dot{X} = f(X) + BU + BF_a \quad (4)$$

with $X \in \Re^n$ is the state vector of the system, $U \in \Re^m$ is the input control vector, and $F_a \in \Re^q$ is the resultant vector of actuator faults related to quadrotor attitude motions, such as:

$$X = \left[x_1, ..., x_6\right]^T = \left[\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}\right]^T \quad (5)$$

Therefore the state space model (4) can be rearranged as follows [15], [16]:

$$\begin{aligned} &S_1 \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a_1 x_4 x_6 + a_2 x_2^2 + a_3 \Omega_r x_4 + b_1 u_1 + b_1 f_{a1} \end{cases} \\ &S_2 \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = a_4 x_2 x_6 + a_5 x_4^2 + a_6 \Omega_r x_2 + b_2 u_2 + b_2 f_{a2} \end{cases} \\ &S_3 \begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = a_7 x_2 x_4 + a_8 x_6^2 + b_3 u_3 + b_3 f_{a3} \end{cases} \end{aligned} \quad (6)$$

with

$$\begin{cases} a_1 = \dfrac{I_y - I_z}{I_x} \; a_2 = -\dfrac{K_{fax}}{I_x} \; a_3 = -\dfrac{J_r}{I_x} \; a_4 = \dfrac{I_z - I_x}{I_y} \\ a_5 = -\dfrac{K_{fay}}{I_y} \; a_6 = \dfrac{J_r}{I_y} \; a_7 = \dfrac{I_x - I_y}{I_z} \; a_8 = -\dfrac{K_{faz}}{I_z} \\ b_1 = \dfrac{l}{I_x} \; b_2 = \dfrac{l}{I_y} \; b_3 = \dfrac{1}{I_z} \end{cases} \quad (7)$$

***Assumption 1:*** The resultants of actuator faults related to attitude motions are assumed to be zero values prior to the faults time and be the constant values after the faults occurs.

$$f_{ai} = \begin{cases} 0 & \text{if } t < T_i \\ f_i^+ & \text{if } t \geq T_i \end{cases}, \; \text{i} \in [1,2,3] \quad (8)$$

where $\{f_1^+, f_2^+, f_3^+\}$ are positive constants.

The problem of trajectory tracking is thus divided in the respective problem for three subsystems: control of roll ($S_1$), pitch ($S_2$), and yaw ($S_3$) motions. Based on backstepping approach, the control design for each subsystem taking into account the resultants of actuator faults related to roll, pitch, and yaw motions, will be carried out in the following subsections in two steps.

### *A. Control of roll motion*

***Step 1:*** For the first step we consider the first tracking-error

$$e_1 = x_1 - x_{1d} \quad (9)$$

Let the First Lyapunov function candidate

$$V_\phi(e_1) = \frac{1}{2}e_1^2 \quad (10)$$

The time derivative of (10) is given by

$$\dot{V}_\phi(e_1) = e_1\dot{e}_1 = e_1(x_2 - \dot{x}_{1d}) \quad (11)$$

The stabilization of $e_1$ can be obtained by introducing a new virtual control $x_2$

$$(x_2)_d = \dot{x}_{1d} - c_1 e_1; \; c_1 > 0 \quad (12)$$

The equation (11) is then

$$\dot{V}_\phi(e_1) = -c_1 e_1^2 \leq 0 \quad (13)$$

***Step 2:*** For the second step we consider the following tracking-error

$$e_2 = x_2 - \dot{x}_{1d} + c_1 e_1 \quad (14)$$

The augmented Lyapunov function is given by:

$$V_\phi\left(e_1, e_2, \tilde{f}_{a1}\right) = \frac{1}{2}\left(e_1^2 + e_2^2 + \tilde{f}_{a1}^2\right); \; \tilde{f}_{a1} = f_{a1} - \hat{f}_{a1} \quad (15)$$

It's time derivative is then:

$$\begin{aligned} \dot{V}_\phi\left(e_1, e_2, \tilde{f}_{a1}\right) &= e_1\dot{e}_1 + e_2\dot{e}_2 + \tilde{f}_{a1}\dot{\tilde{f}}_{a1} \\ &= e_1(-c_1e_1 + e_2) + e_2\left(\dot{x}_2 - \ddot{x}_{1d} + c_1\dot{e}_1\right) + \tilde{f}_{a1}\left(-\dot{\hat{f}}_{a1}\right) \\ &= -c_1e_1^2 + e_2\left(e_1 + a_1x_4x_6 + a_2x_2^2 + a_3\Omega_r x_4 + b_1u_1\right. \\ &\quad \left. - \ddot{x}_{1d} + c_1(-c_1e_1 + e_2) + b_1 f_{a1}\right) + \tilde{f}_{a1}\left(-\dot{\hat{f}}_{a1}\right) \end{aligned} \quad (16)$$

The stabilization of ($e_1$, $e_2$) can be obtained by introducing the following input control

$$\begin{aligned} u_1 = \frac{1}{b_1}\Big(&\ddot{\phi}_d - c_1(-c_1e_1 + e_2) - e_1 - c_2e_2 \\ &- a_1x_4x_6 - a_2x_2^2 - a_3\Omega_r x_4 - b_1\hat{f}_{a1}\Big) \end{aligned} \quad (17)$$

Consequently,

$$\dot{V}_\phi\left(e_1, e_2, \tilde{f}_{a1}\right) = -c_1e_1^2 + e_2\left(-c_2e_2 + b_1\tilde{f}_{a1}\right) + \tilde{f}_{a1}\left(-\dot{\hat{f}}_{a1}\right) \quad (18)$$

In order to compensate the effect of the resultant of actuator faults related to roll motion, an estimated term is introduced. In which, its time derivative is given via an adaptive algorithm as follows

$$\dot{\hat{f}}_{a1}=\alpha_1\tilde{f}_{a1}-\sigma_1 e_2 \qquad (19)$$

It result that

$$\dot{V}_\phi\left(e_1,e_2,\tilde{f}_{a1}\right)=-c_1e_1^2+e_2\left(-c_2e_2+b_1\tilde{f}_{a1}\right)+\tilde{f}_{a1}\left(-\alpha_1\tilde{f}_{a1}+\sigma_1e_2\right)$$

$$=-c_1e_1^2-\begin{pmatrix}e_2 & \tilde{f}_{a1}\end{pmatrix}\begin{pmatrix}c_2 & -b_1\\ -\sigma_1 & \alpha_1\end{pmatrix}\begin{pmatrix}e_2\\ \tilde{f}_{a1}\end{pmatrix} \qquad (20)$$

$$=-c_1e_1^2-E_1^T\Upsilon_1E_1/E_1=\begin{pmatrix}e_2 & \tilde{f}_{a1}\end{pmatrix}^T$$

$c_2$, $\alpha_1$ and $\sigma_1$ are chosen so as to make the matrix $\Upsilon_1$ positive definite, which means that, $\dot{V}_\phi\le 0$.

Let us consider the adaptation law (19) that can be written in the following form

$$\dot{\hat{f}}_{a1}=\alpha_1 f_{a1}-\alpha_1\hat{f}_{a1}-\sigma_1e_2 \qquad (21)$$

As the resultant of actuator faults related to roll motion is unknown, the second equation in (6) will be used to compute its value. Consequently, $f_1$ is given by

$$f_{a1}=\frac{1}{b_1}\left(\dot{x}_2-a_1x_4x_6-a_2x_2^2-a_3\Omega_rx_4-b_1u_1\right) \qquad (22)$$

which leads to

$$\dot{\hat{f}}_{a1}=-\alpha_1\hat{f}_{a1}-\frac{\alpha_1}{b_1}\left(-\dot{x}_2+a_1x_4x_6+a_2x_2^2+a_3\Omega_rx_4+b_1u_1\right)-\sigma_1e_2 \qquad (23)$$

By integration of equation (23) in time, it result that

$$\hat{f}_{a1}(t)=\hat{f}_{a1}(0)+\frac{\alpha_1}{b_1}\left(x_2(t)-x_2(0)\right)+\int_0^t h_1(\tau)d\tau \qquad (24)$$

where $h_1=-\alpha_1\hat{f}_{a1}-\frac{\alpha_1}{b_1}\left(a_1x_4x_6+a_2x_2^2+a_3\Omega_rx_4+b_1u_1\right)-\sigma_1e_2$

As result, the adaptation algorithm of the resultant of actuator faults related to roll motion can be computed without the need of using the time-derivative of $x_2(t)$.

### *B. Control of pitch motion*

***Step 3:*** The tracking-error according to this step is given by :

$$e_3=x_3-x_{3d} \qquad (25)$$

The corresponding Lyapunov function is given by:

$$V_\theta\left(e_3\right)=\frac{1}{2}e_3^2 \qquad (26)$$

The time derivative of (26) is given by

$$\dot{V}_\theta\left(e_3\right)=e_3\dot{e}_3=e_3(x_4-\dot{x}_{4d}) \qquad (27)$$

The stabilization of $e_3$ can be obtained by introducing a new virtual control $x_4$

$$\left(x_4\right)_d=\dot{x}_{3d}-c_3e_3;\ c_3>0 \qquad (28)$$

The equation (27) becomes

$$\dot{V}_\theta\left(s_3\right)=-c_3e_3^2\le 0 \qquad (29)$$

***Step 4:*** For this step we choose the fourth tracking-error

$$e_4=x_4-\dot{x}_{3d}+c_3e_3 \qquad (30)$$

The corresponding Lyapunov function is given by:

$$V_\theta\left(e_3,e_4,\tilde{f}_{a2}\right)=\frac{1}{2}\left(e_3^2+e_4^2+\tilde{f}_{a2}^2\right);\ \tilde{f}_{a2}=f_{a2}-\hat{f}_{a2} \qquad (31)$$

It's time derivative is then:

$$\dot{V}_\theta\left(e_3,e_4,\tilde{f}_{a2}\right)=e_3\dot{e}_3+e_4\dot{e}_4+\tilde{f}_{a2}\dot{\tilde{f}}_{a2}$$

$$=-c_3e_3^2+e_4\left(e_3+a_4x_2x_6+a_5x_4^2+a_6\Omega_rx_2+b_2u_2\right. \qquad (32)$$

$$\left.-\ddot{x}_{3d}+c_3(-c_3e_3+e_4)+b_2f_{a2}\right)++\tilde{f}_{a2}\left(-\dot{\hat{f}}_{a2}\right)$$

The stabilization of ($e_3$, $e_4$) can be obtained by introducing the following input control

$$u_2=\frac{1}{b_2}\left(\ddot{\theta}_d-c_3(-c_3e_3+e_4)-e_3-c_4e_4\right. \qquad (33)$$

$$\left.-a_4x_2x_6-a_5x_4^2-a_6\Omega_rx_2-b_2\hat{f}_{a2}\right)$$

By using the control-law (33) and the equation (32) it comes

$$\dot{V}_\theta\left(e_3,e_4,\tilde{f}_{a2}\right)=-c_3e_3^2+e_4\left(-c_4e_4+b_2\tilde{f}_{a2}\right)+\tilde{f}_{a2}\left(-\dot{\hat{f}}_{a2}\right) \qquad (34)$$

A second estimated term is introduced in input control $u_2$ to compensate the effect of the resultant of actuator faults related to pitch motion, in which it's derivative law is given as

$$\dot{\hat{f}}_{a2}=\alpha_2\tilde{f}_{a2}-\sigma_2e_4 \qquad (35)$$

Consequently,

$$\dot{V}_\theta\left(e_1,e_2,\tilde{f}_{a2}\right)=-c_3e_3^2-\begin{pmatrix}e_4 & \tilde{f}_{a2}\end{pmatrix}\begin{pmatrix}c_4 & -b_2\\ -\sigma_2 & \alpha_2\end{pmatrix}\begin{pmatrix}e_4\\ \tilde{f}_{a2}\end{pmatrix} \qquad (36)$$

$$=-c_3e_3^2-E_2^T\Upsilon_2E_2/E_2=\begin{pmatrix}e_4 & \tilde{f}_{a2}\end{pmatrix}^T$$

The negativity of $\dot{V}_\theta$ is assured, if $c_4$, $\alpha_2$ and $\sigma_2$ are chosen so as to make the matrix $\Upsilon_2$ positive definite.

As the resultant of actuator faults related to pitch motion is also unknown, the fourth equation in (6) will be used to compute its value, which means that

$$\dot{\hat{f}}_{a2}=-\alpha_2\hat{f}_{a2}-\frac{\alpha_2}{b_2}\left(-\dot{x}_4+a_4x_2x_6+a_5x_4^2+a_6\Omega_rx_2+b_2u_2\right)-\sigma_2e_4 \qquad (37)$$

The time integration of the law adaptation (37) is given by

$$\hat{f}_{a2}(t)=\hat{f}_{a2}(0)+\frac{\alpha_2}{b_2}\left(x_4(t)-x_4(0)\right)+\int_0^t h_2(\tau)d\tau \qquad (38)$$

where $h_2=-\alpha_2\hat{f}_{a2}-\frac{\alpha_2}{b_2}\left(a_4x_2x_6+a_5x_4^2+a_6\Omega_rx_2+b_2u_2\right)-\sigma_2e_4$

Consequently, the time-derivative of $x_4(t)$ is not used to compute the estimated of the resultant of actuator faults related to pitch motion in equation (38).

### *C. Control of yaw motion*

***Step 5:*** The tracking-error according to this step is given by :

$$e_5=x_5-x_{5d} \qquad (39)$$

His Lyapunov function is obtained as follows:

$$V_\psi\left(e_5\right)=\frac{1}{2}e_5^2 \qquad (40)$$

And it's time derivative is given by

$$\dot{V}_\psi\left(e_5\right)=e_5\dot{e}_5=e_5(x_6-\dot{x}_{5d}) \qquad (41)$$

The stabilization of $e_5$ can be obtained by introducing a new virtual control $x_6$

$$\left(x_6\right)_d=\dot{x}_{5d}-c_5e_5;\ c_5>0 \qquad (42)$$

The equation (41) is then

$$\dot{V}_{\psi}\left(e_5\right) = -c_5 e_5^2 \le 0 \tag{43}$$

***Step 6:*** For this step we choose the sixth tracking-error

$$s_6 = x_6 - \dot{x}_{5d} + c_5 s_5 \tag{44}$$

The augmented Lyapunov function is chosen by:

$$V_{\psi}\left(e_5, e_6, \tilde{f}_{a3}\right) = \frac{1}{2}\left(e_5^2 + e_6^2 + \tilde{f}_{a3}^2\right);\; \tilde{f}_{a3} = f_{a3} - \hat{f}_{a3} \tag{45}$$

It's time derivative is then:

$$\begin{aligned} \dot{V}_{\psi}\left(e_5, e_6, \tilde{f}_{a3}\right) &= e_5\dot{e}_5 + e_6\dot{e}_6 + \tilde{f}_{a3}\dot{\tilde{f}}_{a3} \\ &= -c_5 e_5^2 + e_6\left(e_5 + a_7 x_2 x_4 + a_8 x_6^2 + b_3 u_3 - \ddot{x}_{5d}\right. \\ &\quad \left. + c_5(-c_5 e_5 + e_6) + b_3 f_{a3}\right) + \tilde{f}_{a3}\left(-\dot{\hat{f}}_{a3}\right) \end{aligned} \tag{46}$$

The stabilization of ($e_5$, $e_6$) can be obtained by introducing the following input control

$$\begin{aligned} u_3 = \frac{1}{b_3}\big(&\ddot{\psi}_d - c_5(-c_5 e_5 + e_6) - e_5 - c_6 e_6 \\ &- a_7 x_2 x_4 - a_8 x_6^2 - b_3 \hat{f}_{a3}\big) \end{aligned} \tag{47}$$

The equation (46) becomes

$$\dot{V}_{\theta}\left(e_5, e_6, \tilde{f}_{a3}\right) = -c_5 e_5^2 + e_6\left(-c_6 e_6 + b_3 \tilde{f}_{a3}\right) + \tilde{f}_{a3}\left(-\dot{\hat{f}}_{a3}\right) \tag{48}$$

The adaptation law of the estimated resultant of actuator faults related to yaw motion is given as

$$\dot{\hat{f}}_{a3} = \alpha_3 \tilde{f}_{a3} - \sigma_3 e_6 \tag{49}$$

It result that

$$\begin{aligned} \dot{V}_{\psi}\left(e_5, e_6, \tilde{f}_{a3}\right) &= -c_5 e_5^2 - \begin{pmatrix} e_6 & \tilde{f}_{a3} \end{pmatrix} \begin{pmatrix} c_6 & -b_3 \\ -\sigma_3 & \alpha_3 \end{pmatrix} \begin{pmatrix} e_6 \\ \tilde{f}_{a3} \end{pmatrix} \\ &= -c_5 e_5^2 - E_3^T \Upsilon_3 E_3 / E_3 = \begin{pmatrix} e_6 & \tilde{f}_{a3} \end{pmatrix}^T \end{aligned} \tag{50}$$

$c_6$, $\alpha_3$ and $\sigma_3$ are chosen so as to make the matrix $\Upsilon_3$ positive definite, which implies the negativity of $\dot{V}_{\psi}$ .

Using the sixth equation in (6) for computing the value of the resultant of actuator faults related to yaw motion in the adaptation law (49), it can be show that

$$\dot{\hat{f}}_{a3} = -\alpha_3 \hat{f}_{a3} - \frac{\alpha_3}{b_3}\left(-\dot{x}_6 + a_7 x_2 x_4 + a_8 x_6^2 + b_3 u_3\right) - \sigma_3 e_6 \tag{51}$$

And the time integration of this law adaptation is given as

$$\hat{f}_{a3}(t) = \hat{f}_{a3}(0) + \frac{\alpha_3}{b_3}\left(x_6(t) - x_6(0)\right) + \int_0^t h_3(\tau)\, d\tau \tag{52}$$

where $h_3 = -\alpha_3 \hat{f}_{a3} - \frac{\alpha_3}{b_3}\left(a_7 x_2 x_4 + a_8 x_6^2 + b_3 u_3\right) - \sigma_3 e_6$

As Consequence, the resulting adaptation algorithm is computed without using of the time-derivative of $x_6(t)$

## IV. Simulation Results

The simulation results are obtained based on real parameters in Table. 1 [10], [11], [24] (see the appendix).

The controller parameters are chosen as follows: $c_i$ =5; with $i \in \{1,\ldots, 6\}$, $\alpha_j$ =1 and $\sigma_j$=0.01; with $j \in \{1, 2, 3\}$.

Two cases are treated to evaluate the performances of the proposed controller.

**Case 1 :** Results without faults

The obtained results are shown in Fig. 2 to Fig.4.

**Case 2 :** Results with actuator faults

In this case, we consider three resultants of actuator faults related to roll, pitch, and yaw motions introduced with 100% of maximum values of inputs control $u_1$, $u_2$, $u_3$ respectively at instants 15s, 20s and 25s. The obtained results are shown in Fig. 5 to Fig. 8.

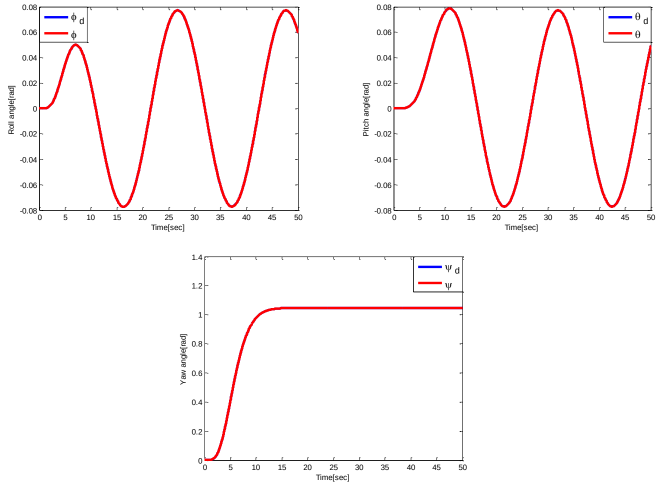


Fig. 2 Tracking simulation results of trajectories along roll ($\phi$), pitch ($\theta$), and yaw ($\psi$) angles, Case 1.

Fig. 2 and Fig. 6 represents the quadrotor attitude tracking, in which the good rotation tracking of quadrotor helicopter is clearly shown for both cases, except a small transient deviations in roll, pitch and yaw motions in case 2 (see Fig. 6) caused by the appearance of actuator faults corresponding to these motions at 15s, 20s and 25s.

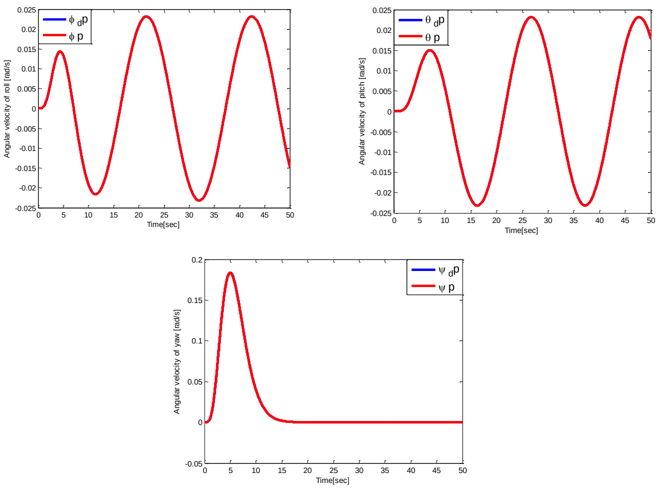


Fig. 3 Tracking simulation results of angular velocities, Case 1.

Fig. 3 and Fig. 7 represents the angular velocities of quadrotor aircraft, the good tracking of the desired velocities is guaranteed for both cases, despite the appearance of a low peaks for the second case (see Fig. 7) in angular velocity of roll, pitch and yaw motions at 15s, 20s and 25s respectively, which means that the robustness of the proposed controller under actuator faults is assured.

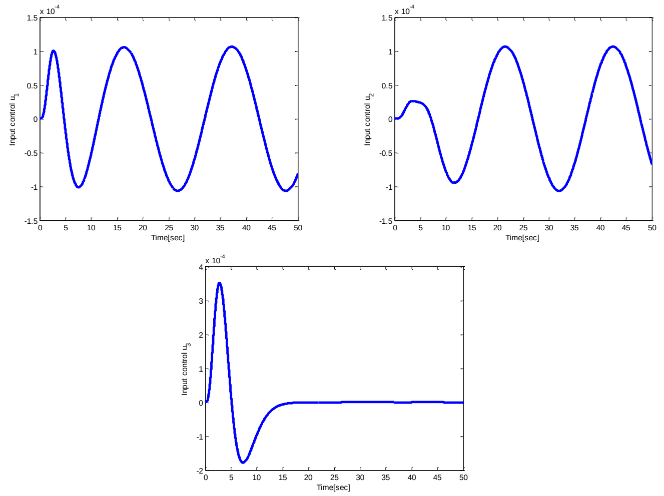


Fig. 4 Simulation results of inputs control ($u_1$, $u_2$, $u_3$), Case 1.

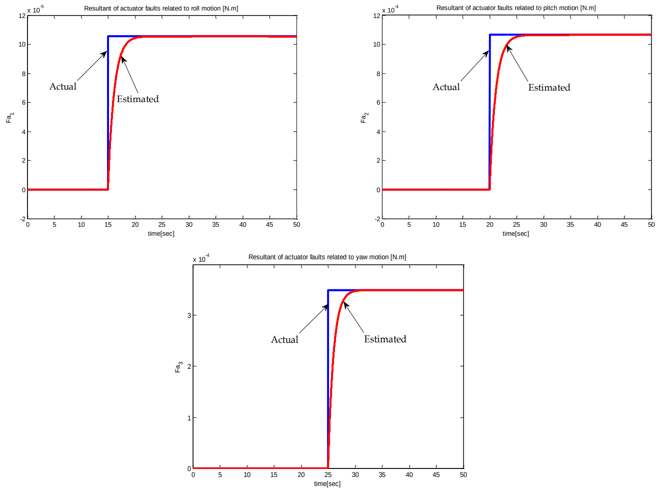


Fig. 5 Simulation results of the Actual and the estimated resultants of actuator faults related to roll, pitch, and yaw motions respectively, Case 2.

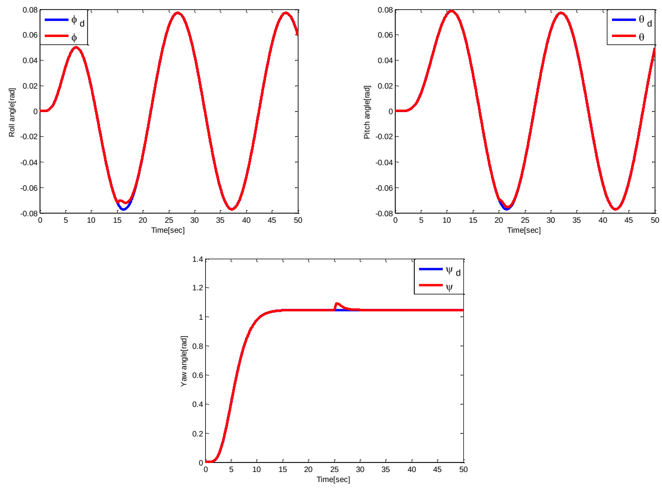


Fig. 6 Tracking simulation results of trajectories along roll ($\phi$), pitch ($\theta$), and yaw ($\psi$) angles, Case 2.

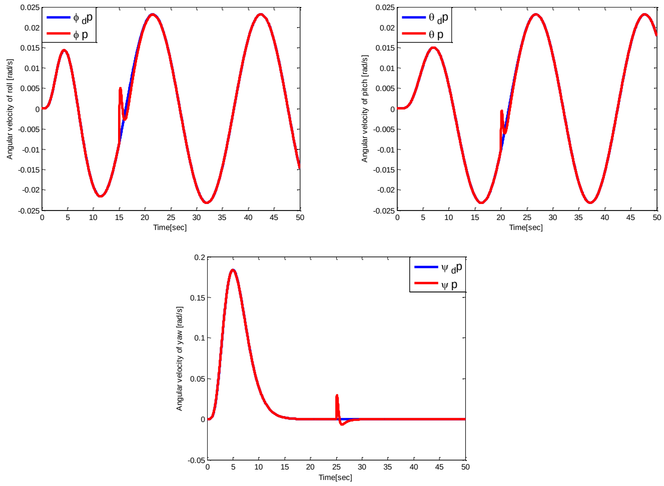


Fig. 7 Tracking simulation results of angular velocities, Case 2.

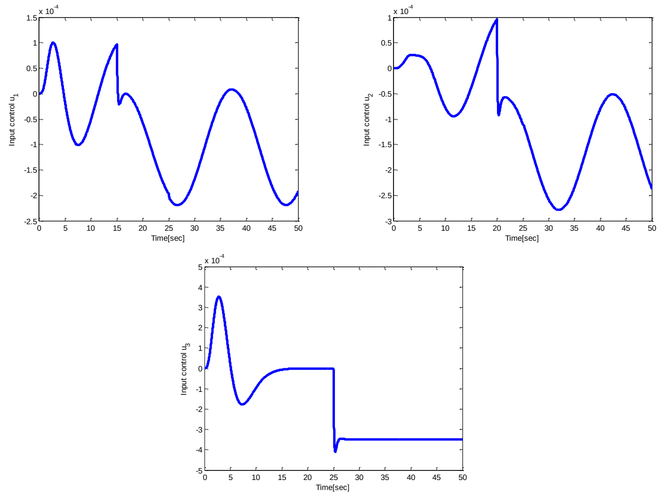


Fig. 8 Simulation results of inputs control ($u_1$, $u_2$, $u_3$), Case 2.

Fig. 4 and Fig. 8 represents the inputs control of quadrotor. From fig. 8, it is clear to see a deviations of input control of roll ($u_1$) at 15s, input control of pitch ($u_2$) at 20s and input control of yaw ($u_3$) at 25s, caused by occurrence of the resultant of actuator faults corresponding to these motions, without any transient pick. We can see also that the obtained input control signals are acceptable and physically realizable.

Fig. 5 represents the resultants of actuator faults related to attitude motions affecting the quadrotor at 15s, 20s and 25s, and these estimated. It can be seen from this figure that the proposed estimator gives a correct estimation for the actual resultants of actuator faults related to attitude motions.

Consequently, it concluded from all simulations presented in this section that the quadrotor performances (summarized by trajectory tracking and stability maintaining) of the closed loop dynamics are assured despite the occurrence of actuator faults affecting its attitude motions, in which, the importance of considering these faults in stability analysis is justified.

## V. Conclusions and Future Works

This paper has successfully demonstrated the application of the robust adaptive-backstepping method to the quadrotor

UAV, in which the actuator faults have been considered. First, the nonlinear failing attitude model of quadrotor which containing the different physics phenomena with the actuator faults affecting the evolution of this system in space was introduced. Then, the stability analysis of the proposed FTC method were derived in detail. Furthermore, The simulation results without and with the consideration of actuator faults were provided, in which the trajectory tracking and the stability maintaining of quadrotor aircraft are assured during the malfunction of these actuators. The implementation of the proposed FTC algorithm on a real prototype will be addressed in the future work.

## APPENDIX

TABLE I
QUADROTOR ATTITUDE MODEL PARAMETERS

| Parameter | Value |
|---|---|
| $m$ | 0,42 *kg* |
| $g$ | 9,806 $m/s^2$ |
| $l$ | 20,5 c*m* |
| $b$ | $2{,}9842 \times 10^{-5}$ *N/rad/s* |
| $d$ | $3{,}2320 \times 10^{-7}$ *N.m/rad/s* |
| $J_r$ | $2{,}8385 \times 10^{-5}$ $kg.m^2$ |
| $I_x$ | $3{,}8278 \times 10^{-3}$ $kg.m^2$ |
| $I_y$ | $3{,}8278 \times 10^{-3}$ $kg.m^2$ |
| $I_z$ | $7{,}1345 \times 10^{-3}$ $kg.m^2$ |
| $K_{fax}$ | $5{,}567 \times 10^{-4}$ *N/rad/s* |
| $K_{fay}$ | $5{,}567 \times 10^{-4}$ *N/rad/s* |
| $K_{faz}$ | $6{,}354 \times 10^{-4}$ *N/rad/s* |
| $k_m$ | $4.3 \times 10^{-3}$ *N.m/A* |
| $k_g$ | 5.6 |
| $R_a$ | 0.67 Ω |

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